# Semantik von Programmiersprachen

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# **IMP**: Syntaktische Kategorien

$$n \in \text{Num}$$
$$x \in \text{Var}$$

$$\begin{aligned}
a \in \text{AExp} &::= n \mid x \\
&\mid a_1 + a_2 \mid a_1 - a_2 \mid a_1 \star a_2 \\
b \in \text{BExp} &::= \texttt{true} \mid \texttt{false} \\
&\mid a_1 = a_2 \mid a_1 \texttt{<=} a_2 \\
&\mid \texttt{not}\ b \mid b_1\ \texttt{and}\ b_2
\end{aligned}$$

$$\begin{aligned}
S \in \text{Stm} &::= \texttt{skip} \\
&\mid x \texttt{ := } a \\
&\mid S_1 \texttt{ ; } S_2 \\
&\mid \texttt{if}\ b\ \texttt{then}\ S_1\ \texttt{else}\ S_2 \\
&\mid \texttt{while}\ b\ \texttt{do}\ S
\end{aligned}$$

# **IMP**: Semantische Kategorien

- Ganze Zahlen $\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$
  - Wert von $n$ $\mathcal{N}[\![n]\!]$
- Wahrheitswerte $\mathbb{B} = \{\mathit{tt}, \mathit{ff}\}$
- Zustände $\Sigma$ Funktionen $\sigma : \mathrm{Var} \to \mathbb{Z}$
  - Variablenzugriff $\sigma(x)$
  - Variablenupdate $\sigma[x \mapsto v]$

$$\sigma[x \mapsto v](x') = \begin{cases} v, & \text{falls } x = x' \\ \sigma(x'), & \text{falls } x \neq x' \end{cases}$$

# **IMP**: Kompositionale Semantik von Ausdrücken

Semantische Funktion $\mathcal{A}[\![-]\!] : \mathrm{AExp} \to (\Sigma \to \mathbb{Z})$

$$\mathcal{A}[\![n]\!]\,\sigma = \mathcal{N}[\![n]\!]$$

$$\mathcal{A}[\![x]\!]\,\sigma = \sigma(x)$$

$$\mathcal{A}[\![a_1 + a_2]\!]\,\sigma = \mathcal{A}[\![a_1]\!]\,\sigma + \mathcal{A}[\![a_2]\!]\,\sigma$$

$$\mathcal{A}[\![a_1 - a_2]\!]\,\sigma = \mathcal{A}[\![a_1]\!]\,\sigma - \mathcal{A}[\![a_2]\!]\,\sigma$$

$$\mathcal{A}[\![a_1 \star a_2]\!]\,\sigma = \mathcal{A}[\![a_1]\!]\,\sigma \cdot \mathcal{A}[\![a_2]\!]\,\sigma$$

# **IMP**: Kompositionale Semantik von Ausdrücken

Semantische Funktion $\mathcal{B}[\![-]\!] : \mathrm{BExp} \to (\Sigma \to \mathbb{B})$

$$\mathcal{B}[\![\texttt{true}]\!]\,\sigma = \mathit{tt}$$

$$\mathcal{B}[\![\texttt{false}]\!]\,\sigma = \mathit{ff}$$

$$\mathcal{B}[\![a_1 \texttt{ = } a_2]\!]\,\sigma = \begin{cases} \mathit{tt}, & \text{falls } \mathcal{A}[\![a_1]\!]\,\sigma = \mathcal{A}[\![a_2]\!]\,\sigma \\ \mathit{ff}, & \text{falls } \mathcal{A}[\![a_1]\!]\,\sigma \neq \mathcal{A}[\![a_2]\!]\,\sigma \end{cases}$$

$$\mathcal{B}[\![a_1 \texttt{ <= } a_2]\!]\,\sigma = \begin{cases} \mathit{tt}, & \text{falls } \mathcal{A}[\![a_1]\!]\,\sigma \leq \mathcal{A}[\![a_2]\!]\,\sigma \\ \mathit{ff}, & \text{falls } \mathcal{A}[\![a_1]\!]\,\sigma > \mathcal{A}[\![a_2]\!]\,\sigma \end{cases}$$

$$\mathcal{B}[\![\texttt{not } b]\!]\,\sigma = \neg\,\mathcal{B}[\![b]\!]\,\sigma$$

$$\mathcal{B}[\![b_1 \texttt{ and } b_2]\!]\,\sigma = \mathcal{B}[\![b_1]\!]\,\sigma \wedge \mathcal{B}[\![b_2]\!]\,\sigma$$

# Transitionssysteme

$(\Gamma, T, \triangleright)$

- Menge der Konfigurationen $\quad \Gamma$
- Menge der terminalen Konfigurationen $\quad T \subseteq \Gamma$
- Transitionsrelation $\quad \triangleright \subseteq (\Gamma \setminus T) \times \Gamma$

- Deterministisches Transitionssystem
  falls $\gamma \triangleright \gamma'$ und $\gamma \triangleright \gamma''$, dann $\gamma' = \gamma''$
- Festgefahrene Konfiguration $\gamma \in \Gamma \setminus T$
  es gibt kein $\gamma'$, sodaß $\gamma \triangleright \gamma'$

# **IMP**: Natürliche Semantik von Anweisungen

Konfigurationen $\langle S, \sigma \rangle \in \mathbf{Stm} \times \Sigma$, $\sigma \in \Sigma$

Terminale Konfigurationen $\sigma \in \Sigma$

Transitionen $\langle S, \sigma \rangle \to \sigma'$

(skip$_{\text{ns}}$) $\langle \texttt{skip}, \sigma \rangle \to \sigma$

(assign$_{\text{ns}}$) $\langle x \texttt{ := } a, \sigma \rangle \to \sigma[x \mapsto \mathcal{A}[\![a]\!]\,\sigma]$

(seq$_{\text{ns}}$) $$\frac{\langle S_1, \sigma \rangle \to \sigma_1 \qquad \langle S_2, \sigma_1 \rangle \to \sigma_2}{\langle S_1 \texttt{ ; } S_2, \sigma \rangle \to \sigma_2}$$

# **IMP**: Natürliche Semantik von Anweisungen

$$(\text{if}^{\text{tt}}_{\text{ns}}) \qquad \frac{\langle S_1, \sigma \rangle \to \sigma_1}{\langle \texttt{if}\ b\ \texttt{then}\ S_1\ \texttt{else}\ S_2, \sigma \rangle \to \sigma_1}, \quad \text{falls } \mathcal{B}[\![b]\!]\,\sigma = \mathit{tt}$$

$$(\text{if}^{\text{ff}}_{\text{ns}}) \qquad \frac{\langle S_2, \sigma \rangle \to \sigma_2}{\langle \texttt{if}\ b\ \texttt{then}\ S_1\ \texttt{else}\ S_2, \sigma \rangle \to \sigma_2}, \quad \text{falls } \mathcal{B}[\![b]\!]\,\sigma = \mathit{ff}$$

$$(\text{while}^{\text{tt}}_{\text{ns}}) \qquad \frac{\langle S, \sigma \rangle \to \sigma' \quad \langle \texttt{while}\ b\ \texttt{do}\ S, \sigma' \rangle \to \sigma''}{\langle \texttt{while}\ b\ \texttt{do}\ S, \sigma \rangle \to \sigma''}, \quad \text{falls } \mathcal{B}[\![b]\!]\,\sigma = \mathit{tt}$$

$$(\text{while}^{\text{ff}}_{\text{ns}}) \qquad \langle \texttt{while}\ b\ \texttt{do}\ S, \sigma \rangle \to \sigma, \quad \text{falls } \mathcal{B}[\![b]\!]\,\sigma = \mathit{ff}$$