IMP: Prozeduren mit dynamischer Bindung

$$p \in \text{Proc}$$
$$P \in \text{ProcDecl} ::= \texttt{proc}\; p\; S\; \texttt{;}\; P \mid \varepsilon$$
$$S \in \text{Stm} ::= \ldots \mid \texttt{begin}\; V\; P\; S\; \texttt{end} \mid \texttt{call}\; p$$

Semantische Kategorien

- Prozedurumgebungen $\mathit{PEnv} = \text{Proc} \rightharpoonup \text{Stm}$
- Konfigurationen $\langle S, \pi, \sigma \rangle \in \text{Stm} \times \mathit{PEnv} \times \Sigma, \quad \sigma \in \Sigma$

Prozedurdeklarationsupdate $\quad upd_P : \text{ProcDecl} \times \mathit{PEnv} \to \mathit{PEnv}$

$$upd_P(\varepsilon, \pi) = \pi$$
$$upd_P(\texttt{proc}\; p\; S\; \texttt{;}\; P, \pi) = upd_P(P, \pi[p \mapsto S])$$

(block$_{ns}$) $$\frac{\langle S, upd_P(P, \pi), upd_V(V, \sigma) \rangle \to \sigma'}{\langle \texttt{begin}\; V\; P\; S\; \texttt{end}, \pi, \sigma \rangle \to \sigma'[\text{dvar}(V) \mapsto \sigma]}$$

(call$_{ns}$) $$\frac{\langle S, \pi, \sigma \rangle \to \sigma'}{\langle \texttt{call}\; p, \pi, \sigma \rangle \to \sigma'}, \quad \text{falls } \pi(p) = S$$

IMP: Prozeduren mit statischer Bindung

$$p \in \text{Proc}$$
$$P \in \text{ProcDecl} ::= \texttt{proc}\ p\ S\ \texttt{;}\ P \mid \varepsilon$$
$$S \in \text{Stm} ::= \ldots \mid \texttt{begin}\ V\ P\ S\ \texttt{end} \mid \texttt{call}\ p$$

Semantische Kategorien

- Prozedurumgebungen $\quad PEnv = \text{Proc} \rightharpoonup (\text{Stm} \times PEnv)$
- Konfigurationen $\quad \langle S, \pi, \sigma \rangle \in \text{Stm} \times PEnv \times \Sigma, \quad \sigma \in \Sigma$

Prozedurdeklarationsupdate $\quad upd_P : \text{ProcDecl} \times PEnv \to PEnv$

$$upd_P(\varepsilon, \pi) = \pi$$
$$upd_P(\texttt{proc}\ p\ S\ \texttt{;}\ P, \pi) = upd_P(P, \pi[p \mapsto (S, \pi)])$$

(block$_{\text{ns}}$) $$\frac{\langle S, upd_P(P, \pi), upd_V(V, \sigma) \rangle \to \sigma'}{\langle \texttt{begin}\ V\ P\ S\ \texttt{end}, \pi, \sigma \rangle \to \sigma'[\text{dvar}(V) \mapsto \sigma]}$$

(call$_{\text{ns}}$) $$\frac{\langle S, \pi_p[p \mapsto (S, \pi_p)], \sigma \rangle \to \sigma'}{\langle \texttt{call}\ p, \pi, \sigma \rangle \to \sigma'}, \quad \text{falls } \pi(p) = (S, \pi_p)$$

IMP: Speicher

Semantische Kategorien

- Speicherzellen $\quad Loc$
- Speicher $\quad Store = Loc \to \mathbb{Z}$
- Variablenumgebungen $\quad VEnv = \mathrm{Var} \to Loc$
- Konfigurationen $\quad \langle S, \upsilon, \varsigma \rangle \in \mathrm{Stm} \times VEnv \times Store, \quad \varsigma \in Store$

Zustandsrekonstruktion $\quad state : VEnv \to Store \to \Sigma, \qquad state\, \upsilon\, \varsigma = \varsigma \circ \eta$

(assign$_{\text{ns}}$) $\quad \langle x := a, \upsilon, \varsigma \rangle \to \varsigma[\upsilon\, x \mapsto \mathcal{A}[\![a]\!](state\, \upsilon\, \varsigma)]$

IMP: Deklarationen mit statischer Bindung

$$
\begin{aligned}
p &\in \text{Proc} \\
V &\in \text{VarDecl} ::= \texttt{var}\ x\ \texttt{:=}\ a\ \texttt{;}\ V \mid \varepsilon \\
P &\in \text{ProcDecl} ::= \texttt{proc}\ p\ S\ \texttt{;}\ P \mid \varepsilon \\
S &\in \text{Stm} ::= \ldots \mid \texttt{begin}\ V\ P\ S\ \texttt{end} \mid \texttt{call}\ p
\end{aligned}
$$

Semantische Kategorien

- $\mathit{Loc} = \mathbb{N}, \qquad \mathit{new} : \mathit{Loc} \to \mathit{Loc}, \quad \mathit{new}\, l = l + 1$
- $\mathit{Store} = (\mathit{Loc} \to \mathbb{Z}) \times (\{\mathit{next}\} \to \mathit{Loc})$
- $\mathit{VEnv} = \text{Var} \to \mathit{Loc}$
- $\mathit{PEnv} = \text{Proc} \rightharpoonup (\mathit{Stm} \times \mathit{VEnv} \times \mathit{PEnv})$
- Konfigurationen $\quad \langle S, \upsilon, \pi, \varsigma \rangle \in \text{Stm} \times \mathit{VEnv} \times \mathit{PEnv} \times \mathit{Store}$

Zustandsrekonstruktion $\quad \mathit{state} : \mathit{VEnv} \to \mathit{Store} \to \Sigma, \quad \mathit{state}\, \upsilon\, \varsigma = (\pi_1\, \varsigma) \circ \eta$

IMP: Deklarationen mit statischer Bindung

Prozedurdeklarationsupdate

$$upd_P : \mathrm{ProcDecl} \times \mathit{VEnv} \times \mathit{PEnv} \to \mathit{PEnv}$$

$$upd_P(\varepsilon, \upsilon, \pi) = \pi$$

$$upd_P(\texttt{proc}\ p\ S\ \texttt{;}\ P, \upsilon, \pi) = upd_P(P, \upsilon, \pi[p \mapsto (S, \upsilon, \pi)])$$

Variablendeklarationsupdate

$$upd_V : \mathrm{VarDecl} \times \mathit{VEnv} \times \mathit{Store} \to \mathit{VEnv} \times \mathit{Store}$$

$$upd_V(\varepsilon, \upsilon, \varsigma) = (\upsilon, \varsigma)$$

$$upd_V(\texttt{var}\ x\ \texttt{:=}\ a\ \texttt{;}\ V, \upsilon, \varsigma) =$$

$$upd_V(V, \upsilon[x \mapsto l], \varsigma[l \mapsto \mathcal{A}[\![a]\!](\mathit{state}\ \upsilon\ \varsigma)][\mathit{next} \mapsto \mathit{new}\ l]) \quad \text{mit } l = \varsigma\ \mathit{next}$$

(block$_{\mathrm{ns}}$) $$\frac{\langle S, \upsilon', upd_P(P, \upsilon', \pi), \varsigma' \rangle \to \varsigma''}{\langle \texttt{begin}\ V\ P\ S\ \texttt{end}, \upsilon, \pi, \varsigma \rangle \to \varsigma''}, \quad \text{falls } upd_V(\upsilon, \varsigma) = (\upsilon', \varsigma')$$

(call$_{\mathrm{ns}}$) $$\frac{\langle S, \upsilon_p, \pi_p[p \mapsto (S, \upsilon_p, \pi_p)], \varsigma \rangle \to \varsigma'}{\langle \texttt{call}\ p, \upsilon, \pi, \varsigma \rangle \to \varsigma'}, \quad \text{falls } \pi(p) = (S, \upsilon_p, \pi_p)$$

IMP: Referenzen

$$r \in \text{RefExp} ::= x \mid \&x \mid \star r$$

$$a \in \text{AExp} ::= n \mid a_1 + a_2 \mid a_1 - a_2 \mid a_1 \star a_2 \mid r$$
$$b \in \text{BExp} ::= \ldots \mid r_1 = r_2$$

Semantische Kategorien

- Speicherzellen Loc
- Speicher $\mathit{Store} = \mathit{Loc} \to \mathit{Loc} \uplus \mathbb{Z}$
- Variablenumgebungen $\mathit{VEnv} = \text{Var} \to \mathit{Loc}$

Semantische Funktionen

- $\mathcal{R}[\![-]\!] : \text{RefExp} \to \mathit{VEnv} \to \mathit{Store} \to (\mathit{Loc} \cup \mathbb{Z} \cup \{\bot\})$

$$\mathcal{R}[\![x]\!]\, \upsilon\, \varsigma = \varsigma(\upsilon(x))$$
$$\mathcal{R}[\![\&x]\!]\, \upsilon\, \varsigma = \upsilon(x)$$
$$\mathcal{R}[\![\star r]\!]\, \upsilon\, \varsigma = \begin{cases} \varsigma(\mathcal{R}[\![r]\!]\, \upsilon\, \varsigma), & \text{falls } \mathcal{R}[\![r]\!]\, \upsilon\, \varsigma \in \mathit{Loc} \\ \bot, & \text{sonst} \end{cases}$$

IMP: Referenzen

- $\mathcal{A}[\![-]\!] : \mathrm{AExp} \to \mathit{VEnv} \to \mathit{Store} \to (\mathbb{Z} \cup \{\bot\})$

$$\mathcal{A}[\![n]\!]\,\upsilon\,\varsigma = \mathcal{N}[\![n]\!]$$

$$\mathcal{A}[\![r]\!]\,\upsilon\,\varsigma = \mathcal{R}[\![r]\!]\,\upsilon\,\varsigma$$

$$\mathcal{A}[\![a_1\ \mathit{bop}\ a_2]\!]\,\upsilon\,\varsigma = \begin{cases} \mathcal{A}[\![a_1]\!]\,\upsilon\,\varsigma\ [\![\mathit{bop}]\!]\ \mathcal{A}[\![a_2]\!]\,\upsilon\,\varsigma, \\ \qquad \text{falls } \mathcal{A}[\![a_1]\!]\,\upsilon\,\varsigma, \mathcal{A}[\![a_2]\!]\,\upsilon\,\varsigma \in \mathbb{Z} \\ \bot, \quad \text{sonst} \end{cases}$$

- $\mathcal{B}[\![-]\!] : \mathrm{BExp} \to \mathit{VEnv} \to \mathit{Store} \to (\mathbb{B} \cup \{\bot\})$

$$\mathcal{B}[\![r_1 = r_2]\!]\,\upsilon\,\varsigma = \begin{cases} (\mathcal{R}[\![r_1]\!]\,\upsilon\,\varsigma = \mathcal{R}[\![r_2]\!]\,\upsilon\,\varsigma), \\ \qquad \text{falls } \mathcal{R}[\![r_1]\!]\,\upsilon\,\varsigma, \mathcal{R}[\![r_2]\!]\,\upsilon\,\varsigma \in \mathit{Loc} \vee \\ \qquad\qquad \mathcal{R}[\![r_1]\!]\,\upsilon\,\varsigma, \mathcal{R}[\![r_2]\!]\,\upsilon\,\varsigma \in \mathbb{Z} \\ \bot, \text{sonst} \end{cases}$$

Natürliche Semantik $\qquad \langle S, \upsilon, \varsigma \rangle \in \mathrm{Stm} \times \mathit{VEnv} \times \mathit{Store}$

(assign$_{\text{ns}}$) $\qquad \langle x := a, \upsilon, \varsigma \rangle \to \varsigma[\upsilon\, x \mapsto \mathcal{A}[\![a]\!]\,\upsilon\,\varsigma]$

IMP: Typen für Referenzen

$\tau \in \mathit{Type} ::= \mathit{int} \mid \mathit{bool} \mid \mathit{void} \mid \&\tau$

Typumgebungen $\Gamma = \{x_1 : \tau_1, \ldots, x_n : \tau_n\}$ mit $\forall 1 \leq i \neq j \leq n \,.\, x_i \neq x_j$

- Zugriff $\Gamma(x) = \tau_i$, falls $x = x_i$
- Erweiterung $\Gamma, x : \tau = \Gamma \cup \{x : \tau\}$, falls $\forall 1 \leq i \leq n \,.\, x \neq x_i$

$(\text{num}_{\text{typ}})$ $\Gamma \vdash n : \mathit{int}$

$(\text{var}_{\text{typ}})$ $\Gamma, x : \tau \vdash x : \tau$

$(\text{ref}^1_{\text{typ}})$ $\Gamma, x : \tau \vdash \& x : \&\tau$ $\qquad$ $(\text{ref}^2_{\text{typ}})$ $\dfrac{\Gamma \vdash r : \&\tau}{\Gamma \vdash \star r : \tau}$

$(\text{arith}_{\text{typ}})$ $\dfrac{\Gamma \vdash a_1 : \mathit{int} \quad \Gamma \vdash a_2 : \mathit{int}}{\Gamma \vdash a_1 \; \mathit{bop} \; a_2 : \mathit{int}}$ mit $\mathit{bop} \in \{+, -, \star\}$

IMP: Typen für Referenzen

(bool$^1_{typ}$) $\Gamma \vdash nop : bool$ mit $nop \in \{\texttt{true}, \texttt{false}\}$

(bool$^2_{typ}$) $$\frac{\Gamma \vdash a_1 : int \quad \Gamma \vdash a_2 : int}{\Gamma \vdash a_1\ bop\ a_2 : bool}$$ mit $bop \in \{<=, =\}$

(bool$^3_{typ}$) $$\frac{\Gamma \vdash b_1 : bool \quad \Gamma \vdash b_2 : bool}{\Gamma \vdash b_1\ \texttt{and}\ b_2 : bool}$$

(bool$^4_{typ}$) $$\frac{\Gamma \vdash b : bool}{\Gamma \vdash \texttt{not}\ b : bool}$$

(bool$^5_{typ}$) $$\frac{\Gamma \vdash r_1 : \tau \quad \Gamma \vdash r_2 : \tau}{\Gamma \vdash r_1 = r_2 : bool}$$

IMP: Typen für Referenzen

(skip$_{\text{typ}}$) $\Gamma \vdash \texttt{skip} : \mathit{void}$

(assign$_{\text{typ}}$) $$\frac{\Gamma \vdash x : \tau \quad \Gamma \vdash a : \tau}{\Gamma \vdash x \texttt{ := } a : \mathit{void}}$$

(seq$_{\text{typ}}$) $$\frac{\Gamma \vdash S_1 : \mathit{void} \quad \Gamma \vdash S_2 : \mathit{void}}{\Gamma \vdash S_1 \texttt{ ; } S_2 : \mathit{void}}$$

(if$_{\text{typ}}$) $$\frac{\Gamma \vdash b : \mathit{bool} \quad \Gamma \vdash S_1 : \mathit{void} \quad \Gamma \vdash S_2 : \mathit{void}}{\Gamma \vdash \texttt{if } b \texttt{ then } S_1 \texttt{ else } S_2 : \mathit{void}}$$

(while$_{\text{typ}}$) $$\frac{\Gamma \vdash b : \mathit{bool} \quad \Gamma \vdash S : \mathit{void}}{\Gamma \vdash \texttt{while } b \texttt{ do } S : \mathit{void}}$$