IMP: Subject Reduction für Referenzen

- Typisierung des Speichers

  (int$_{\text{sto}}$) $\varsigma \vdash v : int$ (ref$_{\text{sto}}$) $\dfrac{\varsigma \vdash \varsigma(l) : \tau}{\varsigma \vdash l : \&\tau}$

- Kompatibilität $\Gamma \rhd (\upsilon, \varsigma)$

  $\varsigma \vdash \upsilon(x) : \&\Gamma(x)$, falls $\Gamma(x)$ definiert

Lemma Gelte $\Gamma \rhd (\upsilon, \varsigma)$.

1. $\Gamma \vdash r : \tau \Rightarrow \varsigma \vdash \mathcal{R}[\![r]\!]\,\upsilon\,\varsigma : \tau$.
2. $\Gamma \vdash a : int \Rightarrow \mathcal{A}[\![a]\!]\,\upsilon\,\varsigma \in \mathbb{Z}$.
3. $\Gamma \vdash b : bool \Rightarrow \mathcal{B}[\![b]\!]\,\upsilon\,\varsigma \in \mathbb{B}$.

Lemma Gelte $\varsigma \vdash l_1 : \&\tau_1 \wedge \varsigma \vdash z : \tau_1$.

$$\forall l_2 \in Loc\,.\,\varsigma \vdash l_2 : \tau_2 \Rightarrow \varsigma[l_1 \mapsto z] \vdash l_2 : \tau_2\ .$$

Satz Gelte $\Gamma \rhd (\upsilon, \varsigma)$.

$$\Gamma \vdash S : void \wedge \langle S, \upsilon, \varsigma \rangle \to \varsigma' \Rightarrow \Gamma \rhd (\upsilon, \varsigma')\ .$$

IMP: Denotationelle Semantik von Anweisungen

Semantische Relation $\mathcal{S}[\![-]\!] : \mathbf{Stm} \to \wp(\Sigma \times \Sigma)$

$$
\begin{aligned}
&\mathcal{S}[\![\texttt{skip}]\!] = \{(\sigma, \sigma) \mid \sigma \in \Sigma\}\\
&\mathcal{S}[\![x \texttt{ := } a]\!] = \{(\sigma, \sigma[x \mapsto \mathcal{A}[\![a]\!]\,\sigma]) \mid \sigma \in \Sigma\}\\
&\mathcal{S}[\![S_1 \texttt{ ; } S_2]\!] = \mathcal{S}[\![S_2]\!] \circ \mathcal{S}[\![S_1]\!]\\
&\mathcal{S}[\![\texttt{if } b \texttt{ then } S_1 \texttt{ else } S_2]\!] =\\
&\qquad\qquad \{(\sigma, \sigma') \mid \mathcal{B}[\![b]\!]\,\sigma = tt \land (\sigma, \sigma') \in \mathcal{S}[\![S_1]\!]\}\\
&\qquad\quad \cup \{(\sigma, \sigma') \mid \mathcal{B}[\![b]\!]\,\sigma = f\!f \land (\sigma, \sigma') \in \mathcal{S}[\![S_2]\!]\}\\
&\mathcal{S}[\![\texttt{while } b \texttt{ do } S]\!] =\\
&\qquad \mu(\lambda W.\quad \{(\sigma, \sigma') \mid \mathcal{B}[\![b]\!]\,\sigma = tt \land (\sigma, \sigma') \in W \circ \mathcal{S}[\![S]\!]\}\\
&\qquad\qquad\qquad \cup \{(\sigma, \sigma) \mid \mathcal{B}[\![b]\!]\,\sigma = f\!f\})
\end{aligned}
$$