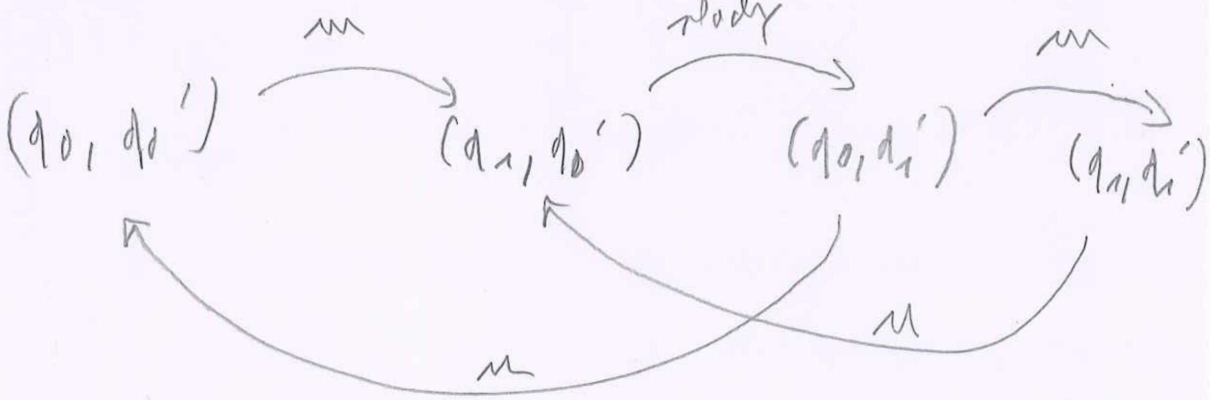
Aufgabe 1

lts(M1) : $q_0 \underset{ready}{\overset{m}{\rightleftarrows}} q_1$, $A_{M1} = \{m, ready\}$

lts(M1) : $q_0' \underset{n}{\overset{ready}{\rightleftarrows}} q_1'$, $A_{M1} = \{ready, n\}$

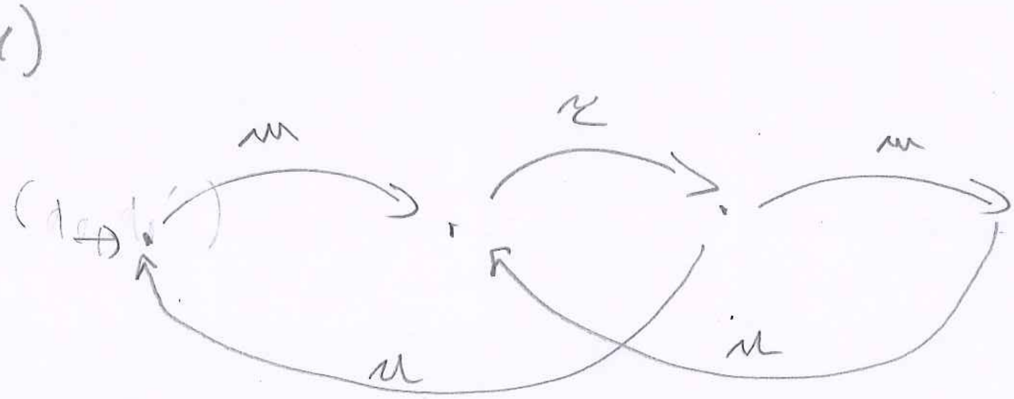
lts(M1 || M1) = Reach(lts(M1) $||_{lts}$ lts(M1))



$A_{M1||M2} = \{m, ready, n\}$

$\Pi S1 = (M1 || M1) \setminus \{ready\}$

lts(S1)



$A_{S1} = \{m, n, \tau\}$

1.2

$lts(M2)$:
$$\begin{tikzcd} q_0 \arrow[r, bend left, "m"] & q_1 \arrow[l, bend left, "\tau"] \end{tikzcd}, \quad A_{M2} = \{m, \tau\}$$

$lts(M2)$:

$lts(U2)$:
$$\begin{tikzcd} q_0' \arrow[r, bend left, "\tau"] & q_1' \arrow[l, bend left, "u"] \end{tikzcd}, \quad A_{U2} = \{\tau, u\}$$

$S2 = (M2 \| U2)$

$lts(S2) := Reach(lts(M2) \|_{ext} lts(U2)) =$

$$\begin{tikzcd}[column sep=large]
(q_0, q_0') \arrow[r, bend left, "m"] \arrow[rrr, bend left=50, "\tau_{(U2)}"] & (q_1, q_0') \arrow[r, bend left, "\tau_{(U2)}"] \arrow[l, bend left, "\tau_{(M2)}"] & (q_1, q_1') \arrow[r, bend left, "\tau_{(M2)}"] \arrow[l, bend left, "u"] & (q_0, q_1') \arrow[l, bend left, "m"] \arrow[lll, bend left=50, "u"]
\end{tikzcd}$$

$A_{S2} = \{m, u, \tau\}$

Aufgabe 3

lts(P1): $a \circlearrowleft q_0 \circlearrowright b$, $A = \{a, b\}$

lts(P2): $a \circlearrowleft q_0' \underset{b}{\overset{\tau}{\rightleftarrows}} q_1'$, $A' = \{a, b, \tau\}$

<u>Beh.</u> $lts(P1) \not\approx lts(P2)$, also $P1 \not\approx P2$

<u>Bew.</u>: !

Annahme: $\exists$ schwache Bisim. $R \subseteq S \times S'$ mit $(q_0, q_0') \in R$ $\Rightarrow$

(2) gilt für (q_0, q_0') $\Rightarrow$

für $q_0' \overset{\epsilon}{\Longrightarrow}_{\Delta'} q_1'$ ex. $p \in \{q_0\}$

mit $q_0 \overset{\epsilon}{\Longrightarrow}_{\Delta} p$ und $(p, q_1') \in R$.

Einzige Möglichkeit:

$p = q_0$ und $q_0 \overset{\epsilon}{\Longrightarrow} q_0$ und $(q_0, q_1') \in R$.

wegen (1) für (q_0, q_1') gilt dann:

Für $q_0 \overset{a}{\Longrightarrow} q_0$ ex. $p' \in \{q_0', q_1'\}$ mit

$q_1' \overset{a}{\Longrightarrow} p'$ $\lightning$. $\square$

lts(P3):

$$\begin{tikzcd} q_0'' \arrow[loop left, "a"] \arrow[r, bend left, "b"] & q_1'' \arrow[l, bend left, "\tau"] \end{tikzcd} \qquad A'' = \{a, b, \tau\}$$

<u>Beh</u>: lts(P1) $\approx$ lts(P3), also P1 $\approx$ P3

<u>Bew</u>:

(a) $\alpha\,$lts(P1) $= \{a, b\} = \alpha\,$lts(P3)

(b) Definiere $R = \{(q_0, q_0''), (q_0, q_1'')\}$

Zeige: R ist eine schwache Bisimulation und $(q_0, q_0'') \in R$ ✓

- Betrachte $(q_0, q_0'') \in R$

(1) 1. Fall: $q_0 \overset{a}{\Longrightarrow}_{\Delta} q_0$

$\exists\; q_0'' \overset{a}{\Longrightarrow}_{\Delta''} q_0''$ und $(q_0, q_0'') \in R$

2. Fall: $q_0 \overset{b}{\Longrightarrow}_{\Delta} q_0$

$\exists\; q_0'' \overset{b}{\Longrightarrow}_{\Delta''} q_1''$ und $(q_0, q_1'') \in R$

(2) →1. Fall: $q_0'' \overset{a}{\Longrightarrow}_{\Delta''} q_0''$

$\exists\ q_0 \overset{a}{\Longrightarrow}_{\Delta} q_0 \leadsto (q_0, q_0'') \in R$

→2. Fall: $q_0'' \overset{b}{\Longrightarrow}_{\Delta''} q_1''$

$\exists\ q_0 \overset{b}{\Longrightarrow}_{\Delta} q_0 \leadsto (q_0, q_1'') \in R$

→3. Fall: $q_0'' \overset{b}{\Longrightarrow}_{\Delta''} q_0''$

$\exists\ q_0 \overset{b}{\Longrightarrow}_{\Delta} q_0 \leadsto (q_0, q_0'') \in R$

Betrachte $(q_0, q_1'') \in R$

1) 1. Fall: $q_0 \overset{a}{\Longrightarrow}_{\Delta} q_0$

$\exists\ \underbrace{q_1'' \overset{a}{\Longrightarrow}_{\Delta''} q_0''} \leadsto (q_0, q_0'') \in R$

mit $q_1'' \xrightarrow{\tau}_{\Delta''} q_0'' \xrightarrow{a}_{\Delta''} q_0''$

2. Fall: $q_0 \overset{b}{\Longrightarrow}_{\Delta} q_0$

$\exists\ q_1'' \overset{b}{\Longrightarrow}_{\Delta''} q_1'' \leadsto (q_0, q_1'') \in R$

mit $q_1'' \xrightarrow{\tau}_{\Delta''} q_0'' \xrightarrow{b}_{\Delta''} q_1''$

(2) 1. Fall: $q_1'' \overset{\epsilon}{\Longrightarrow}_{\Delta''} q_0''$

$q_0 \overset{\epsilon}{\Longrightarrow}_{\Delta} q_0 \leadsto (q_0, q_1'') \in R$

2. Fall: $q_1'' \overset{a}{\Longrightarrow}_{\Delta''} q_0''$

$\exists\ q_0 \overset{a}{\Longrightarrow}_{\Delta} q_0 \leadsto (q_0, q_0'') \in R$

3. Fall: $q_1'' \overset{b}{\Longrightarrow}_{\Delta''} q_1''$

$\exists\ q_0 \overset{b}{\Longrightarrow}_{\Delta} q_0$ und $(q_0, q_1'') \in R$

4. Fall: $q_1'' \overset{b}{\Longrightarrow}_{\Delta''} q_0''$

$(q_1'' \xrightarrow{\tau}_{\Delta''} q_0'' \xrightarrow{b}_{\Delta''} q_1'' \xrightarrow{\tau}_{\Delta''} q_0'')$

$\exists\ q_0 \overset{b}{\Longrightarrow}_{\Delta} q_0$ und $(q_0, q_0'') \in R$ $\square$

$p2 \not\approx p3$

Denn wäre $p2 \approx p3$, dann wäre, wegen $p1 \approx p3$ und weil $\approx$ symmetrisch und transitiv ist, auch $p1 \approx p2$ ↯.