Performance Modelling of Computer Systems
Tutorial 3

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The Geometric Progression

Prove that for all $q \neq 1$ it holds

$$s_n := \sum_{i=0}^{n} q^i = \frac{1 - q^{n+1}}{1 - q} \ .$$

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Especially, this formula implies in the case of $|q| < 1$

$$\sum_{i=0}^{\infty} q^i = \lim_{n \to \infty} \frac{1 - q^{n+1}}{1 - q} = \frac{1}{1 - q} \; .$$

Car repair shop (from the book of W.J. Stewart)

The arrival pattern of cars to the car repair shop follows a Poisson distribution at a rate of four per hour. Let us assume that there is only one mechanic and the service time is exponentially distributed and is on average 12 minutes. What is the ...

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1. $\mathbb{P}(\text{“more than 3 cars are waiting”}) = \sum_{k=4}^{\infty} \pi_k = 1 - \pi_0 - \pi_1 - \pi_2 - \pi_3$. Using $\pi_k = \rho^k(1-\rho)$, $k \geq 0$, we get the probability 0.4096.

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2. $L = \rho/(1-\rho) = 4$.
3. Little's law asserts that $W = L/\lambda = 1/(\mu - \lambda) = 1$ hour.