Exercise 8-1 Bisimilarity

Show that

$$(P|[\{a\}]|Q)\ |[\emptyset]|\ (P|[\{a\}]|Q) \not\sim (P|[\emptyset]|P)\ |[\{a\}]|\ (Q|[\emptyset]|Q),$$

where $P \overset{def}{\equiv} b.P'$, $P' \overset{def}{\equiv} a.P$, $Q \overset{def}{\equiv} c.Q'$ and $Q' \overset{def}{\equiv} a.Q$. (You may write $\|_L$ instead of $|[L]|$ and $\|$ instead of $|[\emptyset]|$, if you like.)

Solution:

- A: $\big(P \parallel_{\{a\}} Q\big) \parallel \big(P \parallel_{\{a\}} Q\big) \xrightarrow{b} \big(P' \parallel_{\{a\}} Q\big) \parallel \big(P \parallel_{\{a\}} Q\big)$
- Case 1) D: $\big(P \parallel P\big) \parallel_{\{a\}} \big(Q \parallel Q\big) \xrightarrow{b} \big(P' \parallel P\big) \parallel_{\{a\}} \big(Q \parallel Q\big)$
- Case 2) D: $\big(P \parallel P\big) \parallel_{\{a\}} \big(Q \parallel Q\big) \xrightarrow{b} \big(P \parallel P'\big) \parallel_{\{a\}} \big(Q \parallel Q\big)$

Without loss of generality, we may assume that we are in the case 1). Then we continue with

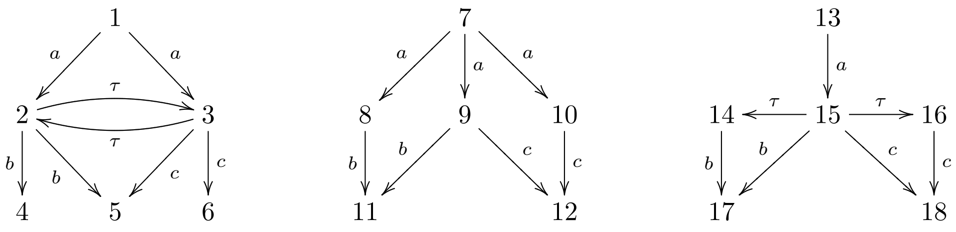
- A: $\big(P' \parallel_{\{a\}} Q\big) \parallel \big(P \parallel_{\{a\}} Q\big) \xrightarrow{c} \big(P' \parallel_{\{a\}} Q\big) \parallel \big(P \parallel_{\{a\}} Q'\big)$
- Case 1.1) D: $\big(P' \parallel P\big) \parallel_{\{a\}} \big(Q \parallel Q\big) \xrightarrow{c} \big(P' \parallel P\big) \parallel_{\{a\}} \big(Q' \parallel Q\big)$
- Case 1.2) D: $\big(P' \parallel P\big) \parallel_{\{a\}} \big(Q \parallel Q\big) \xrightarrow{c} \big(P' \parallel P\big) \parallel_{\{a\}} \big(Q \parallel Q'\big)$

Without loss of generality, we may assume that we are in the case 1.2). Then we continue with

- A: $\big(P' \parallel P\big) \parallel \{a\}\big(Q' \parallel Q\big) \xrightarrow{a} \big(P \parallel P\big) \parallel_{\{a\}} \big(Q \parallel Q\big)$
- D: $\big(P' \parallel_{\{a\}} Q\big) \parallel \big(P \parallel_{\{a\}} Q'\big) \xrightarrow{a} ↯$

Exercise 8-2 Weak Bisimilarity

Given the following LTS.



Prove the following assertions via the bisimilarity game. Use the notation from the solution of 5-3 c).

Solution: Note: The defender can insert arbitrarily many τ into its move. If the attack is a τ, it can answer with the empty transition (staying the the current state). These possibilities all have to be checked for the game to be valid. The attack still only consists of a single action (τ or otherwise).

a) $1 \not\approx 7$

Solution:

(1, 7) Attacker $7 \xrightarrow{a} 10$.

1. Defender $1 \overset{a\tau^n}{\Longrightarrow} 3$.
   (3, 10) Attacker $3 \xrightarrow{\tau} 2$.
   - Defender $10 \Rightarrow 10$.
     (2, 10) Attacker $2 \xrightarrow{b} 4$. $\lightning$
2. Defender $1 \overset{a\tau^n}{\Longrightarrow} 2$.
   (2, 10) Attacker $2 \xrightarrow{b} 4$. $\lightning$

b) $1 \not\approx 13$

Solution:

(1, 13) Attacker $13 \xrightarrow{a} 15$.

1. Defender $1 \overset{a\tau^n}{\Longrightarrow} 3$.
   (3, 15) Attacker $15 \xrightarrow{\tau} 16$.
   - Defender $3 \overset{\tau^{2n}}{\Longrightarrow} 3$.
     (3, 16) Attacker $3 \xrightarrow{\tau} 2$.
     - Defender $16 \Rightarrow 16$.
       (2, 16) Attacker $2 \xrightarrow{b} 4$. $\lightning$
   - Defender $3 \overset{\tau^{1+2n}}{\Longrightarrow} 2$.
     (2, 16) Attacker $2 \xrightarrow{b} 4$. $\lightning$
2. Defender $1 \overset{a\tau^n}{\Longrightarrow} 2$.
   Analog zu (1)

c) $7 \not\approx 13$

Solution:

(7, 13) Attacker $13 \xrightarrow{a} 15$.

- Defender $7 \overset{a}{\Rightarrow} 8$.
  (8, 15) Attacker $15 \xrightarrow{c} 18$. $\lightning$
- Defender $7 \overset{a}{\Rightarrow} 9$.
  (9, 15) Attacker $15 \xrightarrow{\tau} 16$.
  - Defender $9 \Rightarrow 9$.
    (9, 16) Attacker $9 \xrightarrow{b} 11$. $\lightning$
- Defender $7 \overset{a}{\Rightarrow} 10$.
  (10, 15) Attacker $15 \xrightarrow{b} 17$. $\lightning$