Homework till the 23th of May

1. Derive using β-reduction
   - $\mathit{and}\ \mathbf{tt}\ \mathbf{ff} \rightsquigarrow^* \mathbf{ff}$
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2. Let $f := \lambda Z.\lambda s.\mathit{cond}(\llbracket x > 0 \rrbracket s, Z\llbracket y := y + x; x := x - 1\rrbracket s, s)$. Give $f^1\Omega$, $f^2\Omega$, $f^3\Omega$, $f^n\Omega$ and $f^\omega\Omega$, where Ω is as in the lecture.
3. Let us fix a complete partially ordered set $(D, \preccurlyeq)$ and a continuous function $f : D \to D$.
   i) Show that f is monotone, i.e. $f(a) \preccurlyeq f(b)$ for all $a, b \in D$ with $a \preccurlyeq b$.
   ii) Using *i)*, show by beans of induction $\forall i \geq 0\big(f^i(\bot) \preccurlyeq f^{i+1}(\bot)\big)$ and $\forall i \geq 0\big(f^i(\bot) \preccurlyeq x\big)$ for any fixed point x of f.
   iii) Conclude using *i)* and *ii)* the theorem of Knaster-Tarski from the lecture. That is, show that $\sup\{f^i(\bot) \mid i \geq 0\}$ is the least fixed point of f.